College Guild
PO Box 696, Brunswick, Maine 04011

Logic, Puzzles, and Games

Unit 6 of 6

Probability

Welcome back to Logic, Puzzles, and Games.

"What's the chance that…?" We have all asked that question many times throughout our lives. The answer provides us with a degree of certainty and is most often a number that communicates the odds that something will happen: "There is a 75% chance it will rain", "My chances of being a professional athlete are one-in-a-million", "I am six times more likely to catch a virus if I work at an elementary school." In this unit we take a closer look at how these figures are generated and how they help us make optimal decisions when we are playing a game, or just navigating through life.

Glossary of Terms: In alphabetical order

Classical Probability -an approach to determine probability of simple events based on counting the amount of favorable outcomes present and dividing by the total number of possible outcomes.
Denominator -The number on the bottom of a fraction representing the whole.
Dependent Events -events that change each other's chances of happening.
Empirical Probability -also known as experimental probability, a method to determine the probability of any event through the experience of performing many trials, recording the results, and then applying classical probability.
Independent Events -events that do not not affect each other's chances of happening.
Numerator -The number on the top of a fraction representing the part.
Probability -a way to express the likelihood that an event will happen**,** most often represented by a fraction or percentage.
Sample Space -a list of all the ways a sequence of events can occur.
Subjective Probability -an approach to determine probability based purely on instinct, experience, judgement, and opinion.
Theoretical Probability -an approach based on purely applying mathematical formulas to determine the likelihood of an event.
Tree Diagram -a visual tool that shows how a series of possibilities branches out to form a sample space.

"Heads Up"

Probability is a way to communicate how likely an event will happen. Anytime we are wondering about if or when something will occur we are thinking mathematically about chances. Probability can be expressed without numbers by using phrases such as, "I doubt it.", or "I'm not surprised it happened.", or "That sure was lucky!". When we base our expectations on our experience or instinct we are using an approach called **subjective probability**. Even animals guide their decision making through this type of thinking; if a spot on a river proves to be good for fishing, a bear will continue to hunt there, increasing their odds of being well fed.

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Here are some examples of subjective probability being applied:

Ex 1 For flipping a single coin: The probability of the coin landing on HEADS is the same as landing on TAILS, because there are two sides with equal chance of landing on either.

Ex 2 For rolling a pair of dice in a board game: If I rolled a total of 7 three times in a row, my chances of rolling another total of 7 will now be even more likely than before.

Ex 3 For playing the lottery: The chance of losing is much smaller than the chance of winning, since many losers must be necessary to pay for a winner's big prize.

Ex 4 For a gambler at a casino: Winning is more likely to happen on a slot machine when I haven't won in my last twenty bets.

1. Which of the examples above demonstrate that subjective probability can be terribly wrong? Explain what you believe the correct thinking should be.

2. Write a short story where someone, or an animal, uses subjective probability to achieve success due to their correct understanding of chances. Aim to have a minimum of three paragraphs to introduce the characters in the story and get your reader to be genuinely happy for their success.

3. Use subjective probability to express how rare it would be to flip a coin twice and have it land on HEADS twice in a row. Explain how you arrived at your answer.

Mankind has developed three methods of calculating probability to improve their communication on how likely an event is. These approaches are called **Classical**, **Empirical**, and **Theoretical Probability** and although different, each approach results in producing a percent that represents how confident we are that some event will occur.

Throughout the unit we have placed a few fraction facts to assist you with the math you are learning or relearning. If you are a little intimidated by doing math, you are not alone. Just keep an open mind and believe you can do it.

Fast Fraction Fact #1: Fractions represent part of a whole number and the fraction bar represents division. The division symbol actually comes from the way a fraction appears. The top number of a fraction is called the **numerator** and the number on the bottom is called the **denominator**. If you want to write the fraction as a decimal, just divide the numbers. If you want that decimal to become a percent, move the decimal to the right two spaces. You will see this throughout the unit.

Numerator

$$\frac{1}{2} = 1 \div 2 = 0.5 = 50\%$$

Denominator

When we apply **classical probability** we take the number of desired outcomes and divide by the total number of possibilities. Suppose that three children in a room full of thirty were going to be randomly chosen to get a candy bar. If you were one of those children you could use a classical approach to determine the probability of getting the candy bar. You would do this by just making a fraction with the two numbers. You could continue by dividing them into each other and write the answer as a percent between 0% and 100%, or **you can just leave your answer as a fraction**. The letter P below represents the word probability, inside the parenthesis is the event whose probability we are calculating. So we can write

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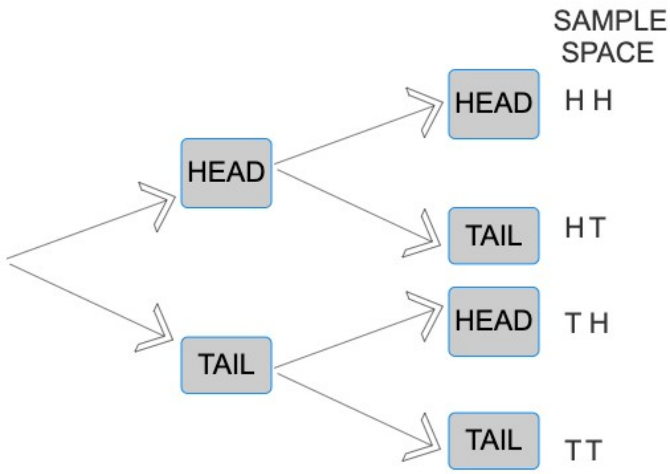
the solution this way:

Number of children who get candy

$$P(winning\ candy) = \frac{3}{30} = 3 \div 30 = 0.10 = 10\%$$

Total number of children

Often building a fraction to express the probability isn't as straight forward as it is in the example above. When there are a series of events occurring we can rely on a visual tool called a **tree diagram** to see the problem more clearly. The tree diagram below shows the different routes that lead to every possible result of flipping a coin twice in a row, the same scenario that you thought about in question 3. This list is known as the **sample space.** It is the organization of the tree diagram that helps us arrive at the information we seek.



Our sample space here has four outcomes for flipping a coin twice in a row:
HEADS HEADS (HH), HEADS TAILS (HT),
TAILS HEADS (TH), and TAILS TAILS (TT).

With all these possibilities listed in the sample space we can apply classical probability to understand how likely it is to get two HEADS in a row.

On the next page is the solution to question 3 based on this sample space.

Number of HEADS HEADS in the sample space

$$P(HH) = \frac{1}{4} = 0.25 = 25\%$$

Total number of possibilities

Another way to calculate a probability is to actually perform the task and record the results. This is an underappreciated technique called **empirical probability.** By actually flipping a coin twice one hundred times we can witness how likely it is to get two HEADS twice a row. We could also program a computer to do the coin flipping for us instead of clumsily flipping and fumbling with all those coin flips. Our answers using this method aren't always perfectly accurate, but are reliably close enough to be useful. Suppose that we did this and got HEADS twice in a row twenty-six times, we could now claim that the solution is:

Number of times we got HEADS HEADS

$$P(HH) = \frac{26}{100} = 0.26 = 26\%$$

Total number of attempts

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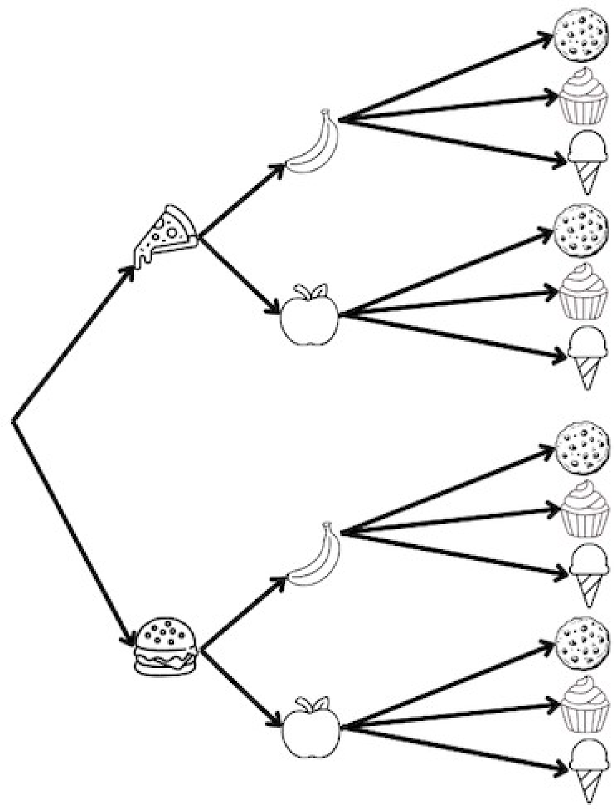

4. Suppose that a cafeteria serves meals that have your choice of entree: pizza or hamburger, the choice of fruit: banana or apple, and the choice of dessert: cookie, cupcake, or ice cream. Use the provided tree diagram to assist you in listing the sample space showing all possible meals. Instead of drawing icons of the food you could just use abbreviations such as B for Burger, and Co for Cookie.

5. If every choice was equally likely, determine the probability that someone will choose a burger with a banana and a cupcake. Write your answer as a fraction.

6. Suppose that each meal also now included a choice of coffee, tea, soda, or milk. Make a list of the sample space of all possible meals that have an entree, fruit, dessert, and drink. Make a tree diagram if you need it. To help form the sample space. How many meals are possible now?

7. A dark closet contains a pair of black shoes and a pair of brown shoes. Using subjective probability, take an educated guess at the probability of randomly reaching in and grabbing a pair of brown shoes. Write your answer as a fraction.

8. Determine the probability of grabbing a pair of brown shoes once again, but this time through classical probability. Draw a tree diagram and sample space to assist you. Write your answer as a fraction.

If all went as predicted you likely did not guess the probability correctly in question 7. This is worth emphasizing. Sometimes probabilities are surprising and are more complicated to determine than we originally think.

"You're on a Roll"

A 17^{th} century French gambler, Antoine Gombaud, had a favorite scam. He bet others that he could roll a SIX at least once in four rolls of a dice. After weeks of running this bet, people caught on that it wasn't a fair bet, so Gombaud invented a more complicated wager, that he could get at least one pair of SIXES in twenty-four rolls of a pair of dice. His discovery, by empirical probability, was that this was a losing bet for him, winning about 47% of the time. Puzzled by gambling questions such as these he turned to legendary mathematicians Blaise Pascal and Pierre de Fermat for help. Together they began the first recorded study of probability. Their goal was to construct mathematical formulas to calculate probability in what is known as **theoretical probability.**

The sample space of all thirty-six possible ways two dice can land when rolled is shown on the next page.

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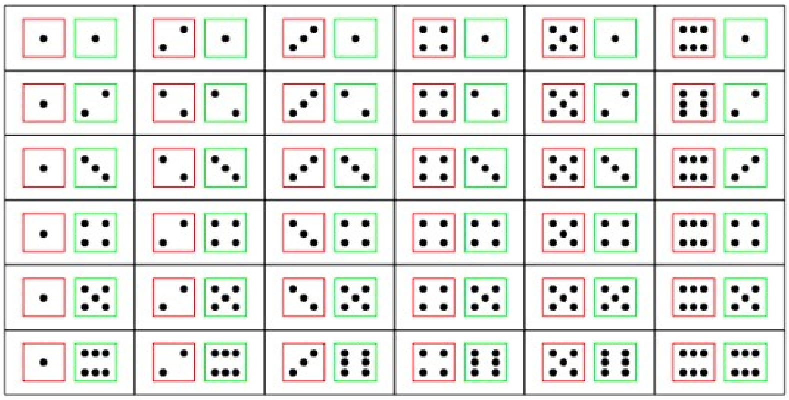
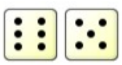


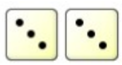
The lowest total is nicknamed "snake-eyes", or double ONES. The highest total, double SIXES, is known to some as "box cars", due to the train car appearance of the dice side by side.

9. Give your own nicknames to the following rolls? Explain your choice for the names.

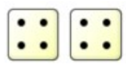
a.



b.



c.



10. Use the sample space above to determine the probability of the following rolls:

a. Double FOURS **b. A total of three** **c. A total of seven** **d. A total of fourteen**

Pascal and Fermat began to analyze results such as these and developed a rule book for calculating probability. To prepare for these we have a few other fraction rules to cover.

Fast Fraction Fact #2: When adding or subtracting fractions the denominators must be the same. You do not add or subtract the denominators, only the numerators. This isn't something you need to know if you have a calculator, but it still is a fact that you will see and use.

Correctly adding and subtraction fractions: $\frac{1}{8}+\frac{3}{8}=\frac{4}{8}$ and $\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$

Incorrectly adding and subtraction fractions: $\frac{1}{8}+\frac{3}{8}=\frac{4}{16}$ and $\frac{3}{4}-\frac{1}{2}=\frac{2}{2}$

When we want to calculate the probability of one OR more events occurring, it will lead to more likely outcomes. So naturally we add the probabilities together to show the odds increasing. Here are two examples that illustrate that adding probabilities make sense when there is an OR in the question.

Ex 1: When rolling a pair of dice: P(total of 10) = 3/36 (about 8%)
P(total of 5) = 4/36 (about 11%)
P(total of 10 OR total of 5) = P(10) +P(5) = 3/36 + 4/36 = 7/36 (about 19%)

Ex 2: When rolling a pair of dice: P(odd number) = 18/36 = 50%
P(greater than 9) = 6/36 (about 17%)
P(odd OR greater than 9) = P(odd) +P(greater than 9) — P(both)
= 18/36 + 6/36 — 2/36 = 22/36 (about 61%)

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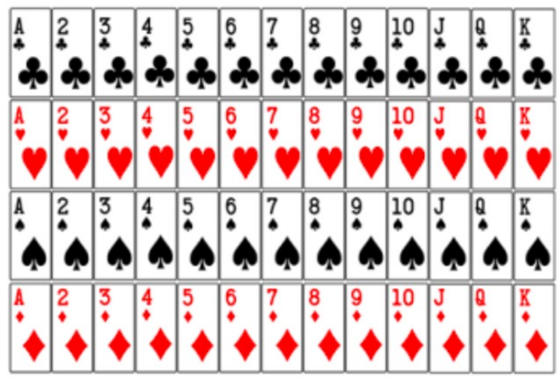
11. Check to see that the answers to Ex1 and Ex 2 are correct by looking back at the sample space on the previous page. Why was subtraction used in Ex 2, but not in Ex 1?

12. Suppose P(owning a dog) = 3/10 and P(owning a cat) = 1/10. Determine the P(owning a dog or a cat) if you can. If it can't be calculated, explain why.

"It's All in the Cards"

It probably isn't surprising that playing cards have been around for centuries and have evolved into the modern deck that we play with today. There are an endless number of games that can be played with these simple cards, from a children's game of Go Fish, to a high stakes Poker championship. It is believed that 70% of magic tricks use cards as the basis of the illusion.

13. If the Aces count as ones, Jacks as eleven, Queens as twelve, and Kings as thirteen, add up the total for the entire deck. Add one more for a Joker card and you end up with a number that should be familiar to you? What is this number and what does it represent outside of the game of cards?



14. Explain why you believe a standard deck of cards has the following features:
a. Two colors: black and red
b. Four suits: Clubs, Hearts, Spades, and Diamonds
c. Fifty-two total cards
d. Pictures with royalty

15. Marvelous Marco the Magician has you pick a card of your choice out of a standard playing card deck with 52 cards in it. You peek at it, and return it to the deck. Marco shuffles the deck and pulls out a card. If there was no trickery involved, determine the following, write your answer as a fraction:
  a. P(it is the same card you picked)
  b. P(that it is not the card you picked)

Fast Fraction Fact #3: Multiplying fractions is much easier than adding or subtracting. Just multiply the numerators together and multiply the denominators together. Of course doing the multiplying without a calculator can be difficult depending on the numbers and your skills. You will see in the questions that follow that this won't stop you from moving forward.

Correctly multiplying fractions: $\frac{1}{8} \times \frac{3}{8} = \frac{3}{64}$ and $\frac{4}{5} \times \frac{3}{4} = \frac{12}{20}$

When we want to calculate the probability of an event occurring AND another event also occurring we multiply the probabilities together. Since each probability is between zero and one, the result of multiplying will be a smaller fraction. Examine the examples that follow.

Ex 1: When rolling a pair of dice:
P(7) = 6/36 = ⅙ (about a 17% chance)
P(rolling a 7 twice in a row) = P(7) x P(7) = ⅙ x ⅙ = $\frac{1}{36}$ (about a 3%)

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Ex 2: When picking cards from a standard deck, replacing the card back to the deck each time:

P(Heart) = 13/52 = 1/4 = 25%

P(Heart three times in a row) = P(Heart) x P(Heart) x P(Heart)

=13/52 x 13/52 x 13/52 = 1/64 (about 1.6%)

Fast Fraction Fact #4: A fraction can be made more simple at times by a process called reducing a fractions. In Ex 1, you see that 6/36 became ⅙ when it was reduced. It can be useful to reduce a fraction but not mandatory. To reduce a fraction find the largest number that can be divided into both the numerator and denominator and divide it into both, like so:

$$\text{Reducing fractions: } \frac{6}{36}\frac{\div 6}{\div 6} = \frac{1}{6} \text{ and } \frac{12}{20}\frac{\div 4}{\div 4} = \frac{3}{5}$$

Ex 3: When picking cards from a standard deck, NOT replacing the cards back to the deck:

P(Heart) = 13/52 = ¼ = 25%

P(Heart three times in a row) =13/52 x 12/51 x 11/50 = 1716/132600 (about 1.3% chance)

These examples show more than the probabilities becoming more rare. In Ex1 and Ex2 the probabilities are **independent**, the probabilities do not influence each other. The calculations in Ex3 illustrate that you must use caution when multiplying probabilities as sometimes they are **dependent**. If a card is not replaced in the deck the next card depends on what happened on what cards have been drawn before, so the probabilities have to change slightly.

16. Calvin doesn't have a calculator but shows you his plan to calculate the following probabilities. For each of the plans tell us if you agree with him or not. If you think he is wrong then show the correct plan. (Notice you do not need to finish the multiplying, just comment on his plans.)

a. P(drawing four aces in a row without placing the cards back in the deck)

Plan: $\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49}$

b. P(drawing the Queen of Spades, rolling an 8 with a pair of dice, and then a tails on a flip)

Plan: $\frac{1}{52} \times \frac{8}{36} \times \frac{1}{2}$

c. P(drawing a Jack, then a Queen, then a King from a deck of cards)

Plan: $\frac{4}{52} \times \frac{4}{52} \times \frac{4}{52} \times \frac{4}{52}$

d. P(holding a Jack and an Ace after two cards are randomly drawn from a deck)

Plan: $\frac{4}{52} \times \frac{3}{51} + \frac{4}{52} \times \frac{3}{51}$

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A positive test for a virus means that there is a high probability that you have the virus, but the testing could be wrong. To clarify, a **false positive** means that the test indicates that you have it, when in fact you don't
A **false negative** means that the test indicates that you don't have it, when in fact you do.

17. In terms of testing for a virus, what are the consequences of a false positive result and what are the consequences of a false negative result?

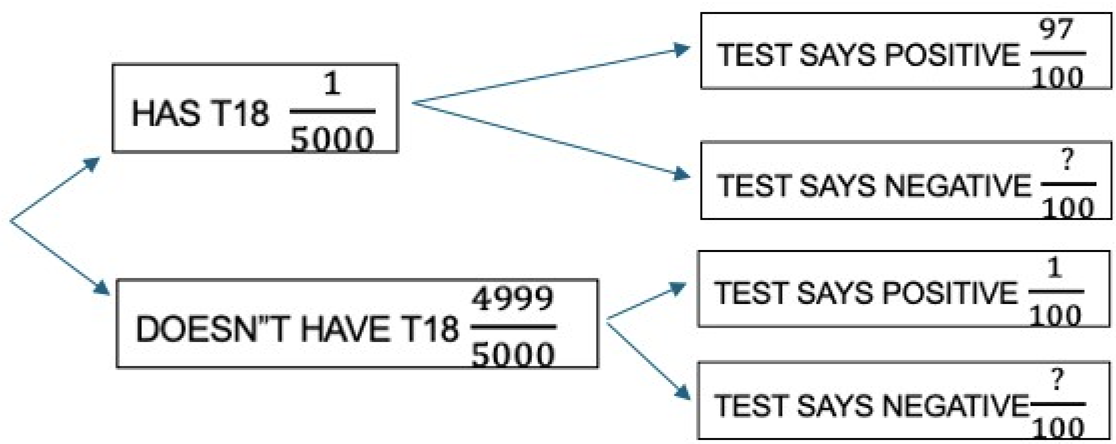
18. If you are a scientist developing a medical test to detect a virus, which result, a false positive or false negative, would you aim for have a lower probability? Explain your choice.

Trisomy 18 (T18) is a rare genetic disorder that severely disrupts a baby's development prior to birth. T18 occurs in only 1 in 5000 pregnancies. A genetic test on the mother's blood can be done to test for T18 in her baby. The probability of a positive test result for a baby with T18 is 97/100. The probability of a positive test result for a baby without T18 is 1/100.

19. Write out a plan on how you would calculate the following probabilities. The tree diagram below could certainly be a big help. If you have the ability or calculator, complete your plan to find the probabilities.. It is fair to mention that parts (c) through (e) are a real challenge so press on and try not to get discouraged.

a. **P(a baby has T18 and the test comes back positive)**
b. **P(a baby does not have T18 and the test comes back positive)**
c. **P(the that a test comes back positive either truthfully OR falsely)**
d. **P(a baby has T18 given that the test comes back positive)**
e. **P(a baby doesn't have T18 given that the test comes back negative)**

Tree Diagram for T18 testing:



Self-Reflection Exercise

20. Did the level of mathematics in this unit inspire you or ruin your taste for math?

21. Name some games that you know of that use dice rolling, or cards. For at least one of these games write a question that could be answered through probability. Answer it if you can.

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