College Guild
PO Box 6448, Brunswick ME 04011

THE NUMBERS GAME
Unit 2 of 4

To be proficient in math, one must know and be able to use the basic number facts; that is, the simple addition and multiplication facts using the numbers 0 through 9. They do not have to be dreary. In fact, some really neat relationships can be found within them. Noticing these relationships makes them a lot more interesting. As elementary school children, we were not shown many of these patterns. In this unit and the next ones, some number oddities are shown which should help you to learn these basic facts – and have some fun at the same time.

PALINDROMIC NUMBERS

Some numbers read the same going left to right as they do going right to left. *56765* is one example. The same is true for words like "radar"; or names like "Anna", "Otto", and "Hannah"; or phrases like "Poor Dan is in a droop", or "Able was I ere I saw Elba." These are called *palindromes*. You have probably heard some. Language lovers like to collect them and love to invent them.

1. Make a list of words and/or names that are palindromes. Next, write down some phrases that you have heard, read, or made up yourself that are palindromes.

There has been a good deal of study on palindromic numbers. It is part of a branch of mathematics called Number Theory. Palindromic numbers are kind of fun to notice on a car's odometer (and there are lots of fun numbers to notice on an odometer), but there's more to them than just the pattern. For example, take the number 742.

Reverse the digits to get a second number and then add them together: 742
+ 247
= 989 It came out palindromic!

2. Try this with the numbers:
- **a. 423**
- **b. 621**
- **c. 238**

As you can see, it doesn't work all the time….

But, try taking it another step with 238. Reverse the digits in your answer, and add that number to your answer. Did you get a palindrome?

In case that was confusing or you didn't get a palindrome, let's do an example together with the number 561, starting on the next page.

Step One:	561	Reverse the 561 to 165 and add the numbers together
	+165	
	= 726	
Step Two:	+ 627	Reverse the answer 726 to 627 and add the numbers together
	= 1353	*(Uh oh…still no palindrome….But, if we go one more step…)*
Step Three:	+ 3531	Reverse the answer 1353 to 3531 and add the numbers together
	= 4884	Bingo! We got a palindrome.

3. Now you try it with the numbers:

a. 43
b. 56
c. 78
d. 35
e. 67

It is easy to check that it will always work for two digit numbers. If the sum of the two digits is less than 10, it is clear that it will work in one step, as you saw when working with 43 and 35. If the sum of the digits is 10, 11, 12, 13, 14, 15, 16, or 18 (as it was with 56, 78, and 67) it works in six steps or less. If the digits add up to 17, however, it can take awhile.

4. What two digit numbers have digits that add up to 17?

5. Why don't we have to worry about instances where the two digits add up to 19?

With three digit numbers, it can get quite big sometimes before it works. It will still work, though!

6. Try it with the numbers:

a. 597
b. 876

Mathematicians (those who like to sit around and add forever or who program computers to do it for them) have found that there are 249 numbers less than 10,000 which do NOT generate palindromes after 100 steps. Aside from these 249 exceptions and 89, all integers produce a palindrome in less than 24 steps. The smallest of these exceptions is 196. If you reverse the digits and add (and have oodles of time), at the end of 230,310 additions you will come out with a palindromic number! The largest palindrome found from integers less than 10,000 is generated by 6,999 in 20 steps.

7. Now, you try it with 89. The process is started for you on the next page. Be sure to keep your numbers in line – ones in the ones place, tens in the tens place, etc. If you get a palindromic number before you have done 24 steps, it means you made an error. Here are some "check points" to help you stay on track:

After the 10^{th} step, you should have 8,872,688.

After the 15^{th} step, you should have 1,317,544,822.

After the 20^{th} step, you should have 93,445,163,438.

The last / 24^{th} step starts 8,813...and it has 13 digits.

			8	9
		+	9	8
	=	1	8	7
	+	7	8	1
	=	9	6	8
	+			
=				
+				
=				

MATH JEOPARDY

Math Jeopardy is a game in which someone says a number, and the responder must ask a question for which this is the answer. If we were playing with names instead of numbers, I might say, "*The answer is George Washington,"* and you would say, "*Who was the first president of the United States?"* In Math Jeopardy, I might say, "The answer is 54." You would have to come up with a question whose answer is 54, such as, "What is 47 + 7?"

Of course, there could be more than one right question for any given answer. If the answer is 54, another perfectly good question is, "What is 60 - 6?" Below, you will be asked to come up with still another question whose answer is 54.

The answers can be refined, restricted, or set up with different rules, such as "the question must be about a multiplication fact," but it's less fun this way. It also means you might not get the chance to come up with a super whizbanger question like, "What is the square root of 2,916?" (That also happens to be 54. ☺) The game has good possibilities for letting a show off be just that without being poisonous. It also has the virtue of teaching that there can be more than one correct response to a math question.

Below are some answers. **You write the questions. Make up <u>several</u> for each answer.** Yes, it's hard work to make up more than 2 or 3 questions for each answer – but you will also find that it becomes more fun and more interesting the more questions you try to write. Good luck!

8. The answer is 17.

9. The answer is 54.

10. The answer is 81.

11. The answer is 256.

12. The answer is _________ ? (You make one up). Now, create some questions to go with it.

13. The answer is _________ ? Create some questions to go with it.

NOTES ON THE FOUR BASIC ARITHMETIC PROCESSES

We are ordinarily taught about addition, subtraction, multiplication, and division separately. It is easier to think of subtraction and division as "un-adding" and "un-multiplying". In subtracting, what we are doing is asking the question, "What must I add to a number to get the given total? For example, 13 – 9 really means "What must I add to 9 in order to get 13?" This is called an *inverse process*. ("Inverse" means something like "opposite of.") Subtraction is the inverse of addition.

Likewise, division is the inverse of multiplication. 18 divided by 3 asks the question, "What must I multiply by 3 to get 18?" So, subtraction and division are inverse processes. We also use the word "inverse" for numbers. For example, negative 8 (-8) is the inverse of positive 8. So, all subtraction problems are merely ones of adding the inverse number, as in this example:

12 – 8 = 4 is exactly the same as 12 + (-8) = 4

For a cool subtraction trick, try the following. Think of a number between 100 and 1000. It should not end in 00, and the difference between the first and last digits should be greater than 1. A number like 842 would be fine. Reverse the digits and subtract the smaller from the larger, like this:

$$\begin{array}{rr} & 842 \\ - & \underline{248} \\ = & 594 \end{array}$$

Now reverse the digits of the answer and add, like this:

$$\begin{array}{rr} & 594 \\ + & \underline{495} \\ = & 1089 \end{array}$$

14. Now, you try it with another number. (Remember, it must be 3 digits, shouldn't end in 00, and the difference between the first and last digits should be greater than 1.) W**hat is your final answer?**

15. Try three more numbers, and write your final answers.

a.
b.
c.

16. Any ideas on why this happens? Take your best shot at finding an explanation.

****

Remember: First names only & please let us know if your address changes

This is how one of our volunteers answered our **Numbers Game Unit 2** questions. Your answers may vary. Years ago, back in the day of the dinosaurs, Reader John got a perfect SAT math score, so we asked him to answer these questions so that our readers and students could compare his answers to theirs. We hope you enjoy them.

1. Palindromes: Kayak, Madam I'm Adam, Pool Loop, Party Trap, "A dog! A panic in a Pagoda!", A Santa at NASA, etc. Can you finish these palindromes? "Pooh Animals …" or "Pull up if …" or "Put Eliot's …" or "Zeus was deified …" There are many others. Ask your family to google "List of Palindromes" and send you many more.

Palindromic Additions (2, 3, 6 and 7) include:

423	621	238	43	56	35	67	78	597	876
+ 324	+126	+832	+34	+65	+53	+76	+87	+795	+678
747	747	1070	77	121	88	143	165	1392	1554
		+0701				+341	+561	+2931	+4551
		1771				484	726	4323	6105
							+627	+3234	+5016
							1353	7557	11121
							+3531		+12111
							4884		23232

```
           89
         + 98
          187
        + 781
          968
        + 869
         1837
       + 7381
         9218
       + 8129
        17347
      + 74371
        91718
      + 81719
       173437
     + 734371
       907808
     + 808709
      1716517
    + 7156171
      8872688
    + 8862788
     17735476
   + 67453771
     85189247
   + 74298158
    159487405
  + 504784951
    664272356
  + 653272466
   1317544822
 + 2284457131
   3602001953
 + 3591002063
   7193004016
 + 6104003917
  13297007933
+ 33970079231
  47267087164
+ 46178076274
  93445163438
+ 83436154439
 176881317877
+ 778713188671
 955594506548
+ 845605495559
1801200002107
+ 7012000021081
8813200023188
```

4. The only digits that add up to 17 are 9 and 8, so 98 and 89 are the answers for #4.

5. The largest single digit is 9. 9+9 is 18. There are no two digits that add to 19. Note that if we use hexadecimal (base 16) numbers, where A stands for 10, B 11, C 12, D 13, E 14 and F 15, we could have A9, B8, C7, D6, E5 and F4 add up to 19, which would be written as 13H. In hex, 19H is equal to one 16 plus 9 ones, or 25. What hexadecimal numbers add up to 19H?

8. Remember, in Jeopardy, you must answer in the form of a question: All of your questions will be different from mine, of course. What day of July was my father born on? What is 11H (hexadecimal) equal to? President Andrew Johnson was what president number?

9. How big is a deck of cards with two Jokers? What is the product of 3 3's and a 2?

10. What is 3^4 (3 to 4^{th} power)? What is the first odd square that is also a power of 4?

11. How many different numbers can be represented in one byte of memory. What is the lowest eighth power of an integer that is not equal to that integer? 2^8. 256 different numbers in a byte? Why? Because there are 8 bits in a byte.

12. 2592001. How many seconds were there in June, 2015. (look up "Leap Second")

13. 31.688087814. How old will you be in years when you are one billion seconds old?

14-15. Answers will vary. 926 – 629 = 297; 297+792 = ??. 604-406 = 198. 198+891 = ??

16. Did you notice that your difference always has a 9 in the middle? What do the first and last digits always add up to? Right, 9.

When you reverse the difference and add the two numbers, you are adding (first plus last digit) hundreds plus (9+9) tens plus (last plus first digit) ones. In other words, our final sum will be 9 hundreds plus 18 tens plus 9 ones. How much is 900 + 180 + 9?

We could write: ABC + CBA = (A+C)*100 + (2*B)*10 + (C+A) = 900 + 180 + 9 = ???
Why don't you try using letters to see if you can figure out why your subtraction caused these numbers to end up with this pattern of 9s: XYZ – ZYX = ??? (The Y's cancel out, so we only need to look at 100(X-Z) – (Z-X) which equals 99X – 99Z.)